

# Evaluation of NCOREL, PAN AIR, and W12SC3 for Supersonic Wing Pressures

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The ability of three recently developed computer programs to predict pressures on a supersonic maneuver wing has been evaluated. The NCOREL program was the only code capable of predicting the nonlinear leeward leading edge supercritical crossflow and resultant shock wave formation at the higher angles of attack. Both linear panel methods, PAN AIR and W12SC3, performed reasonably well at the lower angles of attack. Implementing the freestream axis as the compressibility axis, PAN AIR overpredicted the windward pressures in comparison to the W12SC3 program at higher angles of attack.

## Nomenclature

- $C_p^*$  = pressure coefficient with Carlson correction  
 $M_\infty$  = freestream Mach number  
 $\alpha$  = angle of attack  
 $\delta^*$  = effective flow deflection angle

## Introduction

THE aerodynamicist is continually faced with the problem of selecting the appropriate computer program for a particular requirement. This task is especially risky if the choice involves codes that have not been "calibrated" previously. Thus code comparisons with experimental data are valuable.

Three computer programs were used herein to predict the pressures on a supersonic maneuver wing throughout the moderate to high-lift range at Mach numbers of 1.62 and 2.00. The wing is a NASA sponsored Grumman design that was tested extensively in the Langley Unitary Plan Wind Tunnel.<sup>1</sup> The wing was designed specifically to maintain attached flow at high lift and supercritical cross flow conditions.<sup>2</sup> Hence, unlike most typical supersonic wings, in viscous flow conditions predominate on the upper surface. This makes the wing a prime candidate for evaluating the capability of several existing state of the art inviscid computer codes. The wing was also designed to minimize upper surface cross flow shock formation through a judicious choice of leading edge radius and camber. The baseline wing consisted of an NACA 0004-64 thickness distribution that was modified to reduce the cross-flow shock. Supersonic wings with small leading-edge nose radii typically lead to the formation of strong upper surface cross-flow shocks and, hence upper-surface, shock induced, boundary layer separation and ultimately vortex formation. This leads to the breakdown of inviscid flow conditions on the upper surface and application of any inviscid, linear or nonlinear, theory would be incongruous with this flow. The subject wing has a leading edge radius that is sufficiently large to avoid this problem.

The three computer codes evaluated were W12SC3, PAN AIR, and NCOREL. Both W12SC3 and PAN AIR are linear theory panel codes and NCOREL is a relatively new nonlinear finite difference code. The W12SC3 program is a panel method that is derived from the Woodward codes<sup>3,4</sup> and uses planar wing boundary conditions. Included in the W12SC3 program is the Carlson<sup>5</sup> correction for high alpha nonlinear effects. PAN AIR<sup>6</sup> is a more sophisticated linear theory panel code that, unlike W12SC3, can satisfy more exact surface boundary conditions. Nevertheless both W12SC3 and PAN AIR utilize integral solutions of the linearized potential equation in the form of distributed singularities on a surface panel. NCOREL,<sup>7</sup> on the other hand is a finite difference numerical program that solves the full nonlinear potential equation in a spherical coordinate system using a fully implicit marching technique. NCOREL allows for the development of the bow shock wave, as well as the embedded cross flow shock, which evolves due to the formation of transonic cross flow regions.

Detailed pressure measurements were obtained on the "demonstration wing" at stations transverse to the freestream, Fig. 1. This is a particularly good choice for a highly swept supersonic wing since most of the variation occurs in the transverse plane where the appearance of embedded shock waves can be readily detected. The three computer programs mentioned previously are used to predict the detailed pressure measurements at four stations on the demonstration wing at angles of attack from 6 to 14 deg.

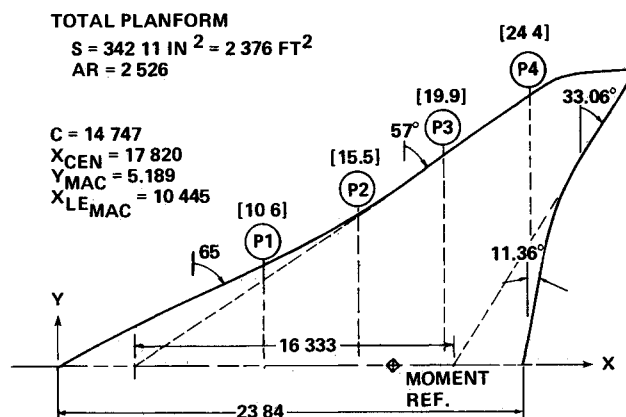


Fig. 1 Demonstration wing model and pressure data systems

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## Description of the Computer Codes

### W12SC3

The W12SC3 linear theory panel method<sup>8</sup> combines and extends elements of the Woodward I<sup>3</sup> and Woodward II<sup>4</sup> codes. Although the most useful elements of both codes have been combined, along with a number of additional features, the baseline code for the development effort was the Woodward B 00 code obtained by Grumman from the NASA Langley Research Center in November 1976. The W12SC3 code consists of a combination of source and vortex panel singularity distributions. The vortex singularities are distributed in either a constant or a piecewise linear fashion streamwise and are piecewise constant spanwise. The source singularities are constant on body panels and piecewise linear streamwise on wing panels.

The W12SC3 program applies linear theory panel methods to find the solutions for wing-body configurations. The user may either calculate wing on body effects on the arbitrary body model (as in Woodward II) or specify that they be calculated on an interference shell that approximates the actual body shape (as in Woodward I).

During the execution of several W12SC3 options, the linearly varying vortex panels used exclusively in Woodward II are replaced by constant strength vortex panels. When this occurs, no extra singularities are used at supersonic trailing edges, and control point locations are fixed at 95% panel chord (85% for subsonic Mach numbers). These changes are necessary for the implementation of the design and optimization options (see Ref. 9), and for cases in which wing-on-body effects are calculated on an interference shell. In addition, constant pressure panels improve the analysis results for wings with supersonic leading edges. Constant pressure panels were used in the solutions presented in this paper. It should be emphasized, however, that the W12SC3 program can reproduce Woodward II pressure distributions when desired.

A modification to the linear theory calculation of pressures on a supersonic lifting surface proposed by Carlson<sup>5</sup> is also computed. The "Carlson correction" is used in conjunction with linear theory to locally account for nonlinear effects. It is intended primarily for high Mach numbers (3.5) and large angles of attack (10-20 deg). The correction is based on the use of shock-expansion relationships to represent the nonlinear variation of pressures with surface slopes in two dimensional flow.

Carlson defines an effective deflection angle  $\delta^*$  as the local surface slope (with respect to the freestream) plus an equivalent turning angle due to three dimensional aerodynamic interference. Prandtl-Meyer expansion relationships are used to relate this equivalent turning angle to the difference between local perturbation velocities calculated by linear theory with and without interference. The pressure coefficient  $C_p^*$  is then evaluated as a function of  $\delta^*$  using two dimensional shock expansion relationships.

As originally formulated, the Carlson correction did not reproduce the linear theory values for small and moderate angles of attack. The formulation was extended at Grumman by Rosen<sup>10</sup> to correct this problem. By incorporating the exact Mach number expansion relationship it was found that the Carlson correction would blend smoothly with linear theory results, without resorting to any empirical corrections.<sup>11</sup> This formulation is termed the "rationalized Carlson correction" and is incorporated in the W12SC3 code.

A problem in implementing this correction in W12SC3 arose when it was discovered that the spanwise velocity variation on the surface agreed with linear theory for a flat delta wing, but was displaced by a nonzero velocity at the centerline. The implementation of the Carlson correction, therefore, is combined with an approximate adjustment to the W12SC3 spanwise velocity. For moderate angles of attack, the resulting error in the spanwise velocity leads to only small effects on the Carlson pressure calculation.

### PAN AIR

PAN AIR is a higher order panel method developed by Magnus and Epton<sup>6</sup> for the solution of linearized potential flows about arbitrary configurations. A more complete description of the method is available in Refs. 12 and 13.

The principal features that distinguish the technique from lower order panel methods are: the incorporation of logically independent networks, the use of higher order singularities ensuring continuity of doublet strength and geometry, and the possibility of various boundary condition specifications.

The configuration is represented by a distribution of source and doublet singularities. The singularities may be placed on the actual configuration surface, or may be used to represent components such as wings in linearized "thin wing" fashion by a single surface. A panel method is used for the solution. Here the configuration surface is divided into networks. A network is defined as a smooth portion of the configuration that subsequently has been divided into panels, each panel representing some source and doublet distribution. A linear source and quadratic doublet distribution over the panels are defined in terms of the values of the singularities at the centers of each panel and neighboring panels by a system of spline type polynomials. Boundary conditions are applied at discrete points associated with each network. Each network is logically independent in that it contributes as many equations as unknowns to the overall boundary value problem. The required integrals are evaluated in closed form and a resulting set of linear equations is solved for the required singularity strength parameters. Once this is accomplished, the potential and velocity fields are known. The pressure field then can be calculated from an appropriate pressure-velocity relationship, and forces and moments calculated by surface pressure integration.

To insure sufficient accuracy and stability for supersonic flow solutions, continuity of both the doublet strength and geometry must be maintained. This was necessary because such discontinuities introduce infinite singularities in velocity which do not decay with distance. A panel system with all contiguous edges was obtained by dividing the basic four corner nonplanar panel into eight triangular subpanels. A quadratic doublet distribution is applied over each triangular subpanel in such a way that the doublet strength is continuous at panel edges, leading to a nine-parameter spline for the complete panel.

Several investigators<sup>14,16</sup> have commented on the need to use velocity boundary conditions at supersonic speeds. Recently Melnik and Mason<sup>17</sup> have demonstrated that mass flux boundary conditions as used in PAN AIR neglect a term that becomes increasingly important with increasing Mach number. All results presented in this paper use the velocity boundary condition formulation, with the option  $IPOT=0$ , which ensures that velocities are computed with velocity influence coefficients, and the second order pressure formulation.

### NCOREL

The approach used in NCOREL consists of casting the nonconservative full potential equation in a spherical coordinate system. Conformal mappings are then used, if necessary, to generate a grid in the spherical cross flow plane capable of resolving typically large gradients in the vicinity of the wing leading edge. The three dimensional full potential equation written in a spherical coordinate system reduces identically to the conical two-dimensional equation at  $R=0$ . The primary advantage to this formulation is that a three dimensional computation can be initiated automatically with a conical flow at the apex using the same set of equations. Other supersonic marching codes (e.g., see Refs. 18 and 19) require either a separate conical computation or another code to generate starting plane data. Unlike other supersonic methods utilizing nonlinear equations, NCOREL is a fully implicit marching technique that is not hindered by CFL

constraints. The only constraint in the marching direction is that due to geometric accuracy. Hence, a three-dimensional wing computation can be achieved in 10 to 50 steps, depending upon accuracy requirements and geometric variation in the marching direction.

The radial spherical coordinate is used as the marching direction. This is an absolute necessity for a method oriented toward wing flowfields at low supersonic Mach numbers. Other marching methods that utilize the axial Cartesian coordinate as the marching direction very quickly encounter subsonic axial Mach numbers below Mach 2 for wings that are not highly swept. Explicit marching codes that use conformal mappings also typically encounter difficulty for wing geometries at low supersonic Mach numbers because the mesh size in the vicinity of the leading edge tends to diminish greatly. The vanishing mesh size and CFL condition in combination greatly restrict the step size in the marching direction, making the computation impractical.

In NCOREL, finite difference solutions to the non-conservative full potential equation are obtained at each spherical cross-flow station commencing at the apex,  $R=0$ , with a conical solution for the geometry obtained at  $R=\Delta R$ . Transonic type-dependent relaxation techniques form the basis for the numerical cross-flow plane solutions. The current finite difference equations are second-order accurate in elliptic cross-flow regions and first-order accurate in the hyperbolic cross-flow regions and spherical marching direction. A rotated difference scheme<sup>20</sup> is used in the supersonic cross-flow region. The type dependency is coupled to the character of the cross-flow Mach number on the spherical surface. The character of the cross-flow terms switch from elliptic to hyperbolic as the cross-flow Mach number exceeds sonic conditions. The numerical technique allows for the capture of both embedded crossflow and bow-type shocks. In Ref. 7, the bow shock can also be fitted isentropically as a boundary. It was found that fitting in the bow shock increased the accuracy of the technique further. As has been shown in Ref. 7, NCOREL has exhibited a high level of success for flows over isolated bodies and wings.

### Geometric Representation

In an attempt to evaluate the PAN AIR code's sensitivity to panel arrangement and density, three separate panel models were developed. A 15 chordwise by 12 spanwise model with panels concentrated at the leading edge, a 15 by 12 cosine spaced model, and a 22 by 16 cosine spaced model, were initially developed. The computed pressure coefficients were practically unchanged for the three panel models, as shown in Fig. 2. Therefore, the standard 15 by 12 model was used for all subsequent comparisons since the heavy panel concentration at the leading edge should help to detect leading-edge compression. Panels were also clustered inboard to define the wing balance housing, thus including the body effect on the results. The wing was divided into five paneling networks. Collectively, these networks describe the outer (wetted) surfaces of the configuration under study and the wake it generates. The upper and lower surfaces comprised two networks. A "tip cap" network was used to avoid a singularity at the wing tip where the upper and lower surfaces meet. Because the trailing edge was of finite thickness (0.2%), two wake networks were used corresponding to the upper surface wake and the lower surface wake, respectively. The wake networks extended approximately 40 chord lengths aft of the wing.

The W12SC3 paneling was based on the analytic definition of the wing. It consisted of 30 chordwise panel edges spaced in a half-cosine distribution, and 20 evenly spaced spanwise panel edges. The aerodynamic solution then was extracted from this model for 20 evenly spaced rows chordwise and 19 evenly spaced rows spanwise for a total of 380 panels.

For supersonic flows, the W12SC3 planar boundary condition option was used, and therefore, the wing geometry was approximated by a camber line with a mean thickness distribution. The body balance housing was modeled as part of the wing mean thickness solution. Although the program can use either linearly varying or constant pressure singularities, the constant singularities were used since these have been shown<sup>9</sup> to provide better results at supersonic speeds.

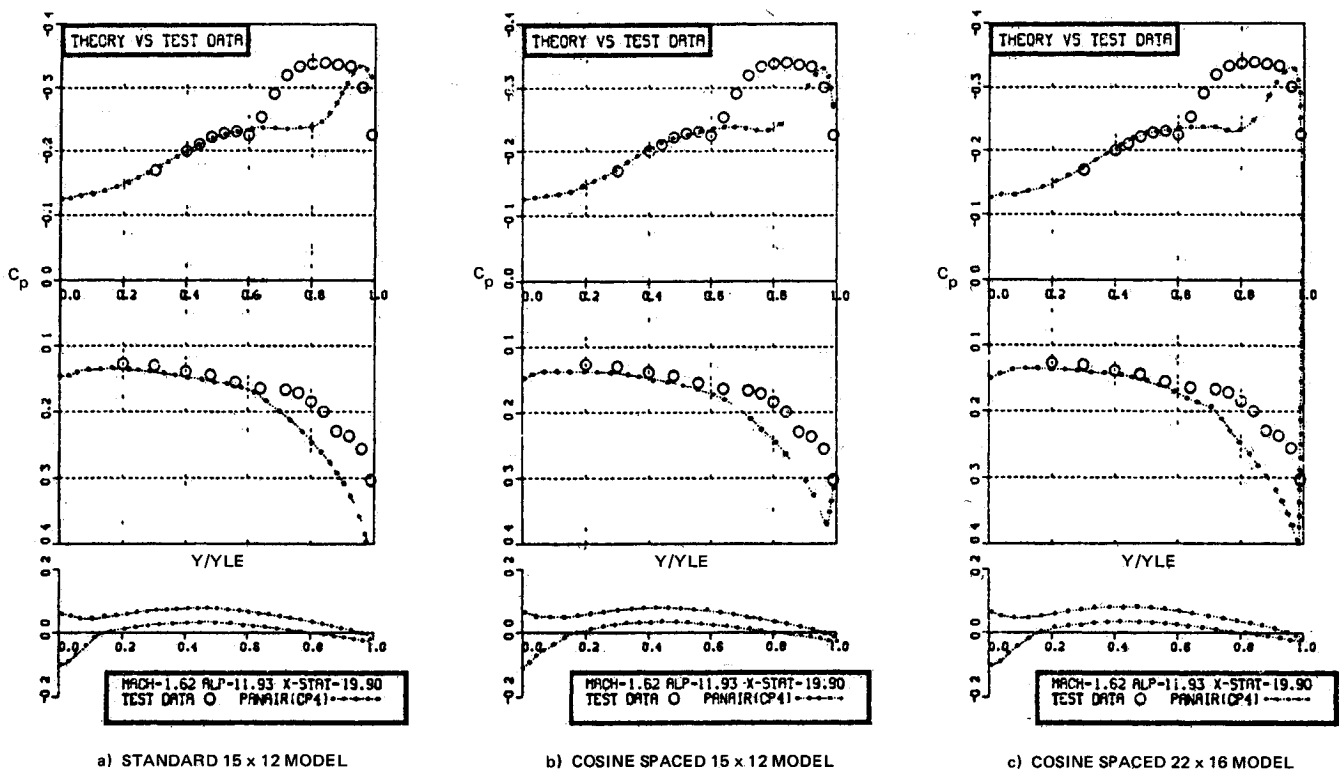


Fig. 2 Computed spanwise pressure distributions for three PAN AIR panel models.

The NCOREL program does not use panels but rather body node points. For all of the comparisons shown, the NCOREL program used approximately 1800 body node points to represent the configuration.

Since the experimental results were for spanwise pressure distributions, the PAN AIR calculated pressures, which are output at panel control points, were interpolated to correspond to the experimental data locations. The W12SC3 code had almost 4 times as many locations where pressure coefficients were calculated, and NCOREL had 20 times the number. Thus, the W12SC3 results were not enhanced by interpolation. The NCOREL results were interpolated directly from the radial spherical data onto the four spanwise stations without any enhancement.

Figure 3 shows a comparison of benchmark running times for the three codes. Although the W12SC3 code ran con-

siderably faster than PAN AIR, the PAN AIR program's solution accuracy was relatively independent of the number of panels (i.e., 360 to 1100). This is not expected to be the case for the W12SC3 program. For a large number of body node points (i.e., > 1000) the NCOREL program was considerably faster than PAN AIR. The 360-panel PAN AIR model was chosen for comparison as a result of both computational economy and insensitivity of results with panel density. The NCOREL fine grid model was used because these results were already available even though a coarser grid could have been used.

## Results

Comparisons are shown between the three theories at  $\alpha = 7.92, 9.92$  and  $11.92$  deg for  $M_\infty = 1.62$ , and  $\alpha = 11.8$  deg for  $M_\infty = 2.0$ , at three of the four span stations shown in Fig. 1.

The NCOREL code predicts the small supercritical (i.e., transonic) cross-flow region developing on the upper surface at  $M_\infty = 1.62$ ,  $\alpha = 7.92$  deg, Fig. 4. Moving aft, the spanwise section becomes thinner, and a small shock develops on the upper surface, Figs. 4b and 4c. This is predicted by NCOREL; the two linear methods cannot predict this nonlinear effect. The PAN AIR results follow the NCOREL prediction outside the leading-edge region. The leading-edge singularity typical of linear theory is clearly exhibited by the W12SC3 results. PAN AIR, which uses higher order singularities and satisfies boundary conditions on the actual wing surface, gives qualitatively better predictions in the leading-edge region.

At a higher angle of attack (Fig. 5) a more pronounced nonlinear supercritical cross-flow region and shock is indicated by NCOREL and the test data. The PAN AIR results cannot predict the nonlinear cross-flow shock. Surprisingly, the post-shock pressures predicted by PAN AIR correlate closely with NCOREL. On the lower surface, the PAN AIR prediction differs considerably from the test data, as well as from NCOREL and W12SC3. W12SC3 generally agrees with PAN AIR on the upper surface, with the exception of the leading-edge region, where W12SC3 exhibits the standard linear theory singularity. On the lower surface, W12SC3

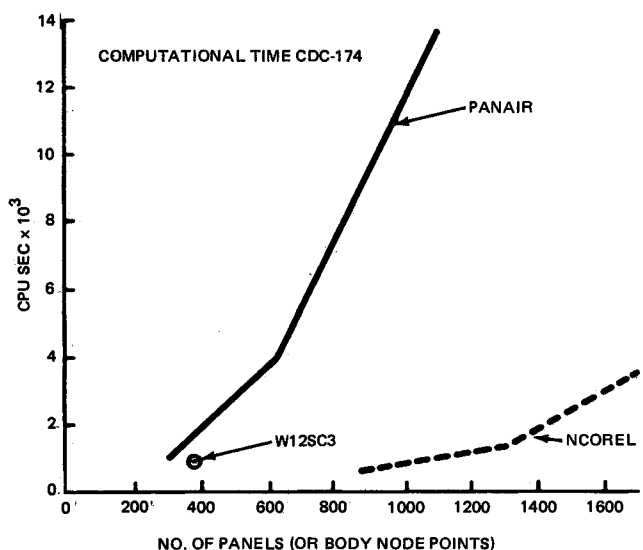


Fig. 3 Comparison of benchmark computer run times for various wing models.

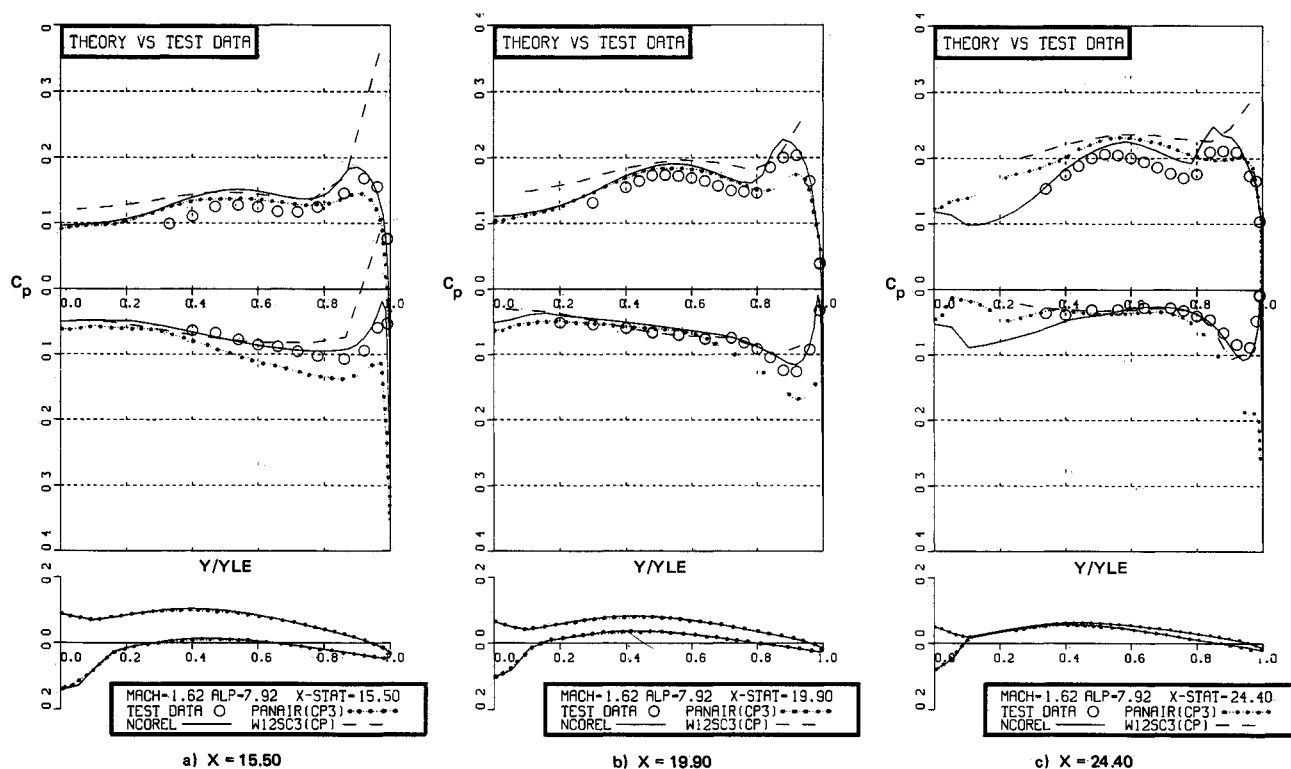


Fig. 4 Comparison of NCOREL, PAN AIR, and W12SC3 spanwise surface pressure distributions with test data at  $M_\infty = 1.62$ ,  $\alpha = 7.92$  deg.

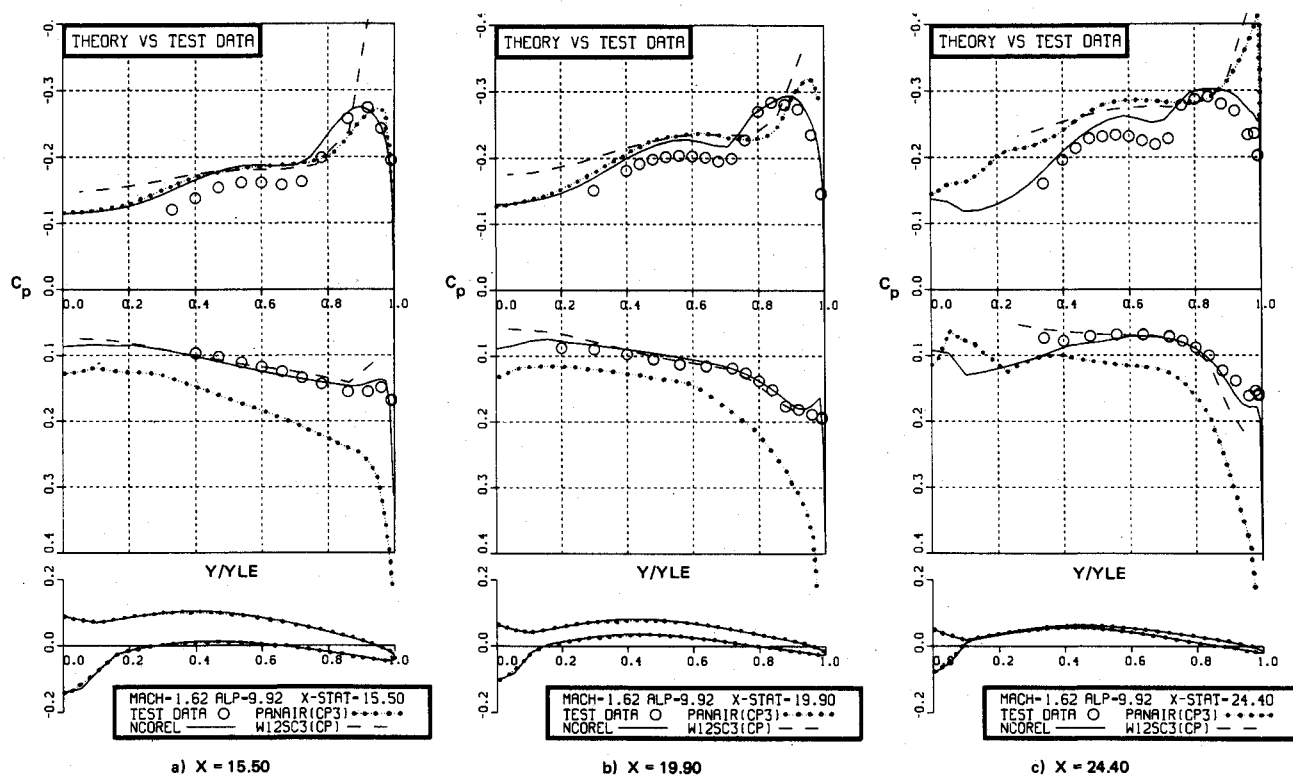


Fig. 5 Comparison of NCOREL, PAN AIR, and W12SC3 spanwise surface pressure distributions with test data at  $M_\infty = 1.62$ ,  $\alpha = 9.92$  deg.

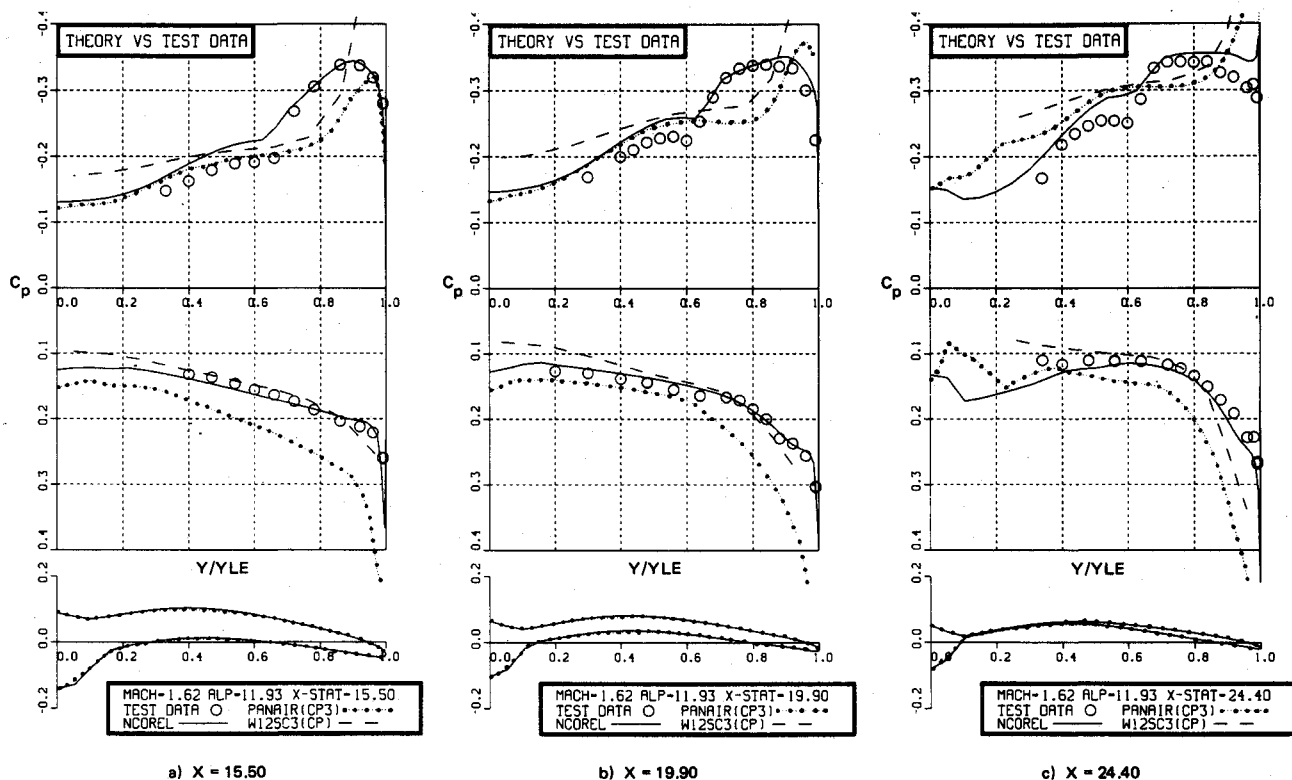


Fig. 6 Comparison of NCOREL, PAN AIR, and W12SC3 spanwise surface pressure distributions with test data at  $M_\infty = 1.62$ ,  $\alpha = 11.93$  deg.

provides good predictions, in contrast to the PAN AIR results.

The significant differences between PAN AIR and W12SC3 (both linear methods) for the lower surface pressures are probably due to the different choice of compressibility axis used by the two codes. The W12SC3 linearizes about the configuration "x axis," while for the PAN AIR runs, the freestream axis was chosen, as is recommended. At the higher angles of attack, the proximity of the Mach cones to the lower

surface in PAN AIR may have distorted the lower surface pressures.<sup>21</sup> This may also explain the better agreement on the upper surface inboard of the supersonic region between PAN AIR and NCOREL. To validate this explanation, the PAN AIR program should be rerun with the compressibility axis aligned with the configuration x axis.

The  $\alpha = 11.93$  deg case is shown in Fig. 6. For this case the nonlinear cross-flow extends over about 40% of the upper surface. The importance of including the nonlinear cross-flow

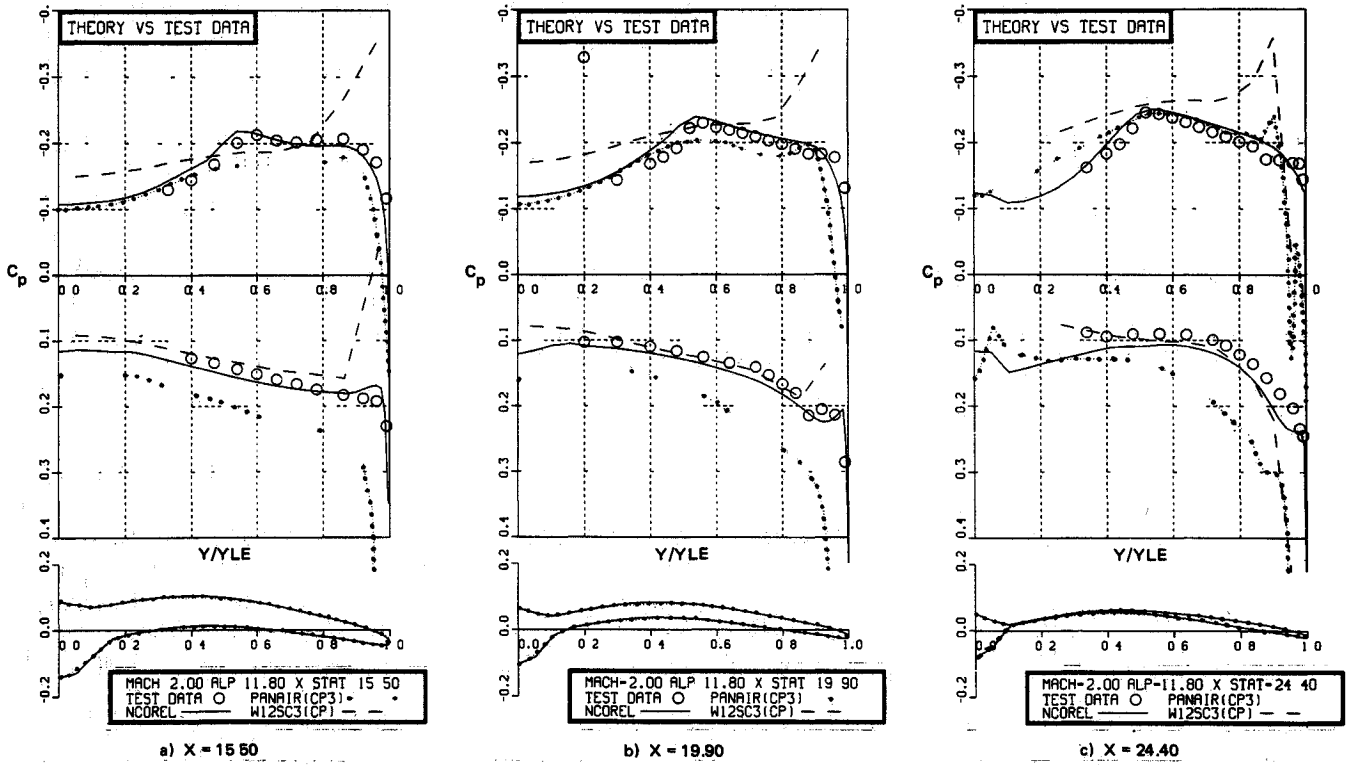


Fig. 7 Comparison of NCOREL, PAN AIR, and W12SC3 spanwise surface pressure distributions with test data at  $M_\infty = 2.0$ ,  $\alpha = 11.80$  deg

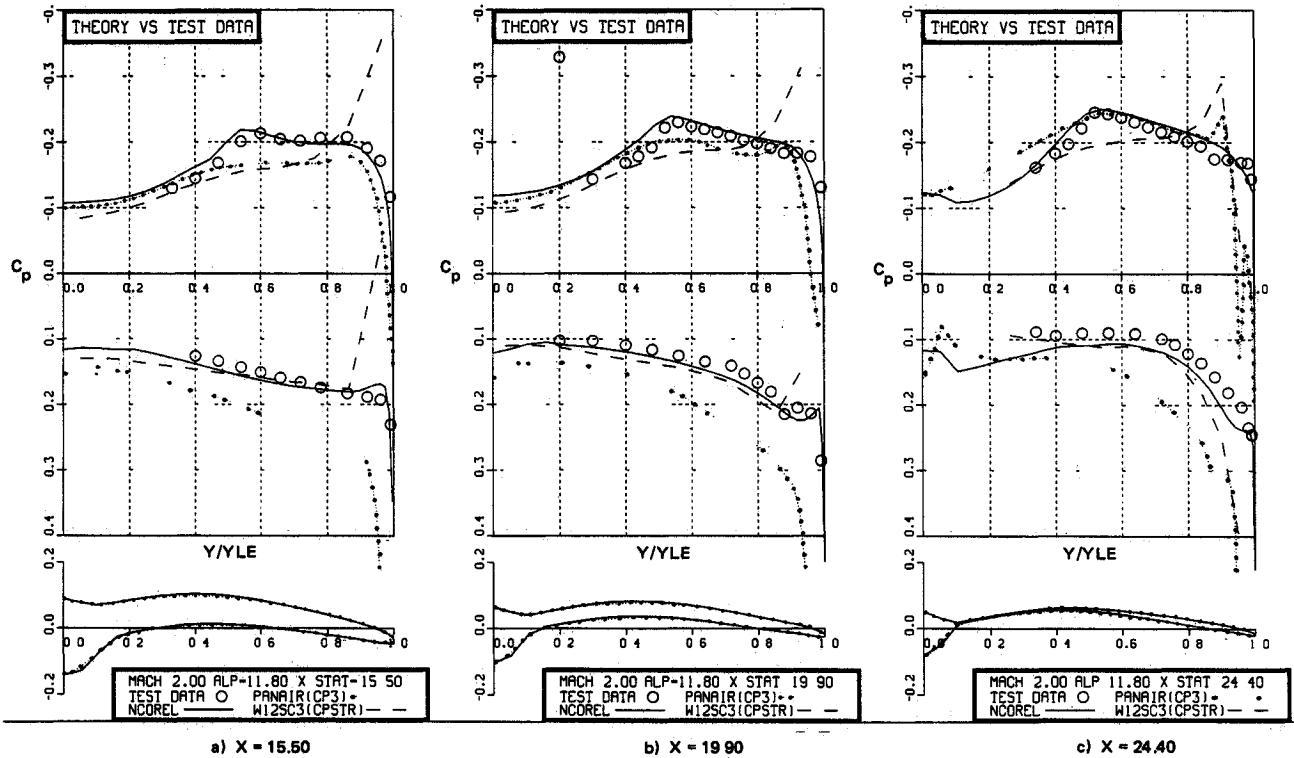


Fig. 8 Comparison of NCOREL, PAN AIR, and W12SC3 (Carlson correction) spanwise surface pressure distributions with test data at  $M_\infty = 2.0$ ,  $\alpha = 11.80$  deg

terms in the governing equations becomes clear. For NCOREL, the agreement is generally good, with essentially minor differences between theory and experiment. At  $X=15.5$  and  $19.9$ , the supercritical cross-flow regions are predicted almost exactly, with the shock jump under-predicted, and the largest disagreement in the pressure level occurring inboard of the cross-flow shock. At  $X=24.4$  the pressure plateau is not predicted quite as well, but it still provides good estimates of the location of the cross-flow

shock. The calculation provides results over the entire span, even though a portion of this region is actually off the trailing edge in the wake. The calculation assumes a rigid wake which extends to the balance housing, and the abrupt emergence of the balance housing causes the irregular predictions inboard of about  $0.3$  semispan. The leading-edge pressure spikes seen in the predictions are not evident in the data. At this station the spanwise section is extremely thin, and the resolution of the grid (60 points around the surface) may not be sufficient.

near the leading edge. Neither linear theory code can provide details of the local pressure distribution. Once again, PAN AIR shows a considerable discrepancy on the lower surface.

At  $M_\infty = 2$ ,  $\alpha = 11.80$  deg, the PAN AIR code does better for upper surface pressures than W12SC3, whereas the W12SC3 code does better on the lower surface, as shown in Fig. 7. Both linear codes do not predict the pressures as well as the nonlinear NCOREL program.

Since for  $M_\infty = 2$ ,  $\alpha = 11.80$  deg, linear theory is of questionable validity, the Carlson correction was applied to the W12SC3 results, as shown in Fig. 8. Although the relative magnitude of the  $C_p$  comparisons change, qualitatively the results were not improved substantially. It would be interesting to apply the Carlson correction to the PAN AIR program.

### Conclusions

The NCOREL code has been shown to be superior to two linear theory methods for the prediction of pressures on a supersonic wing at high angle of attack. This is particularly true for cases where there is a strong cross flow shock on the upper surface. Even for the lower angle of attack cases the NCOREL results indicate that the additional terms in the governing equation and the resolution available at the leading edge using the Joukowski mapping are important.

In general, the PAN AIR and W12SC3 codes provide comparable results, with the PAN AIR code as applied showing better correlation on the upper surface, and W12SC3 results in better agreement with test data on the lower surface. For the low lift and Mach number cases where nonlinear effects are minimal, both linear codes are in good agreement with the test data.

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